LEIBNITZ TEST

(B.Sc.-II, Paper-III)

Group- B

Sudip Kumar

Assistant Professor,

Department of Mathematics

Sachchidanand Sinha College

Aurangabad, Bihar

Leibnitz test

DEFINITION :- (Alternating series) – A series whose terms are alternately positive and negative is called alternating series.

Example :→ ① $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\cdots\cdots\infty$

② $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\frac{1}{2^4}-\cdots\cdots\infty$

③ $1-2+3-4+\cdots\cdots\infty$

Leibnitz test (on alternating series)

Statement :→

The alternating series

$a_1-a_2+a_3-a_4+\cdots\cdots$ (where each $a_n>0$)

is convergent if

(i) $\{a_n\}$ is a monotonic decreasing sequence

i.e. $a_n \geq a_{n+1}$, for all n.

(ii) $\lim_{n\to\infty} a_n = 0$.

Proof :→

Let S_n = sum of first n terms of the given series.

$$\therefore S_{2n} = a_1-a_2+a_3-a_4+\cdots\cdots+a_{2n-1}-a_{2n}$$

$$=(a_1-a_2)+(a_3-a_4)+\cdots\cdots+(a_{2n-1}-a_{2n})$$

≥ 0 (from condition (i))

Again

$$S_{2n} - S_{2n-2} = a_{2n-1} - a_{2n} \geq 0 \; ; \; (\text{by (i)})$$

$\therefore \; S_{2n-2} \leq S_{2n}$, for all n.

$\therefore \; S_2 \leq S_4 \leq S_6 \leq \cdots\cdots\cdots$

$\therefore$ (S_{2n}) is a m.i sequence.

Also,

$$S_{2n} = a_1 - (a_2 - a_3) - (a_4 - a_5) - \cdots\cdots$$
$$\cdots\cdots - (a_{2n-2} - a_{2n-1}) - a_{2n}$$

$\therefore \; S_{2n} \leq a_1$

$\therefore$ $\{S_{2n}\}$ is m.i. sequence and bounded above.

$\therefore$ $\{S_{2n}\}$ is convergent to l (say).

$$\therefore \lim_{n\to\infty} S_{2n+1} = \lim_{n\to\infty} (S_{2n} + a_{2n+1})$$
$$= l + \lim_{n\to\infty} a_{2n+1}$$
$$= l + 0 \quad (\text{by (ii)})$$
$$= l$$

$\therefore \lim_{n\to\infty} S_n = l$.

$\therefore$ The series is convergent.

Example ①:- show that the series

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+ \cdots\cdots$ is convergent.

Solution:→

$\because$ The given series is an alternating series.

$\because$ The sequence $\{\frac{1}{n}\}$ is monotonic decreasing

and $\lim_{n\to\infty} \frac{1}{n} = 0$.

~~The~~ $\therefore$ By Leibnitz test.

The series is convergent.

Example ②:→ show that the series

$1-\frac{1}{5}+\frac{1}{9}-\frac{1}{13}+ \cdots\cdots$ is convergent.

Proof:→

$\because$ The series is an alternating series of the term $a_1 - a_2 + a_3 - a_4 + \cdots$

Where each $a_n > 0$ and $a_n = \frac{1}{4n-3}$.

$$\therefore a_{n+1} = \frac{1}{4n+1}$$

$$\therefore a_n > a_{n+1}, \text{ for all } n.$$

Also, $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{4n-3} = 0$

$\therefore$ The series satisfies all the conditions of leibnitz' test.

Hence the series is convergent.

Thank you